MATH-817 Advanced Functional Analysis

Credit Hours: 3-0
Prerequisite: None

Course Objectives: This course presents functional analysis from a more advanced perspective. The main objective are to 1) understand the classic results of Functional Analysisincluding Zorn's Lemma and Hahn-Banach Theorem, 2) understand the basic concepts of Fixed Point Theory, 3) know and understand the topics on approximation theory.

Previous Knowledge: A student who wishes to opt this course is recommended to have a previous knowledge of elementary functional analysis including Metric Spaces, Normed Spaces, Banach Spaces, Inner Product Spaces and Hilbert spaces. Furthermore, student is required to have a good command on elementary linear algebra.

Core Contents: Fundamental Theorems for Normed and Banach Spaces, Banach Fixed PointTheorem and its applications, Applications of Banach Fixed Point Theorem, Approximation Theory.

Detailed Course Contents: Fundamental Theorems for Normed and Banach Spaces: Zorn's Lemma, Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector Spaces and Normed Spaces, Adjoint Operator, Reflexive Spaces, Category Theorem, Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Open Mapping Theorem, Closed Linear Operators. Closed Graph Theorem.
Further Applications: Banach Fixed Point Theorem: Banach Fixed Point Theorem, Application of Banach's Theorem to Linear Equations, Applications of Banach's Theorem to Differential Equations, Application of Banach's Theorem to Integral Equations.
Approximation Theory: Approximation in Normed Spaces, Uniqueness, Strict Convexity, Uniform Approximation, Chebyshev Polynomials, Approximation in

Hilbert Space

Course Outcomes: This course is specially designed for students who want to choose functional analysis and fixed point theory as their specialty. On successful completion of this course, the students will:

Be able to work with fundamental concepts in functional analysis.

Have a grasp of formal definitions and rigorous proofs in functional analysis. Be able to apply abstract ideas to concrete problems in analysis.

Be aware of applications of basic techniques and theorems of functional analysis in other areas of mathematics, such as fixed point theory, approximation theory, and the theory of ordinary differential equations.

Text Book: Erwin Kreyszig, Introductory Functional Analysis with Applications, Wiley; First edition 1989.

Reference Books:

J. B. Conway. A Course in Functional Analysis. Springer-Verlag , New York, 1985.

George Bachman, Lawrence Narici, Functional Analysis, Dover Publications; 2nd edition, 1998.

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1, 1.3, 1.4, 1.6	**Review:** Metric spaces, Open set, Closed set, Cauchy sequence, Complete metric spaces,
2	2.2,3.1,3.2	**Review:** Normed spaces, Banach spaces, Inner product spaces, Hilbert spaces
3	4.1,4.2	Zorn's Lemma, Hahn-Banach Theorem
4	4.3, 4.5	Hahn- Banach Theorem for complex vector spaces and Normed Spaces, Adjoint Operator
5	4.6	Reflexive spaces
6	4.7	Category Theorem, Uniform Boundedness Theorem
7	4.8	Strong and Weak Convergence
8	**Mid Semester Exam**	
9	4.9	Convergence of sequences of Operators and functionals
10	4.12	Open Mapping Theorem
11	4.13	Closed Linear Operators, Closed Graph Theorem
12	5.1	Banach Fixed Point Theorem
13	5.2, 5.3	Applications of Banach's Theorem to Linear Equations and Differential Equations
14	5.4	Applications of Banach's Theorem to Integral Equations
15	6.1, 6.2	Approximation I Normed Spaces, Uniqueness, Strict Convexity
16	6.3	Uniform Approximation, Chebyshev Polynomial
17		Review
18	**End Semester Exam**	